**Notes: Week of Sept7.Fall2012**

Course website: [www.cis.syr.edu/~sueo/cis275](http://www.cis.syr.edu/~sueo/cis275)

**Quick summary of slides (still look at slides for more reference)**

**A set – collection of elements**

**x ∈ A : x is an element of the set A**

**x ∉ A : x is not an element of the set A**

**Common Sets:**

ℤ - Set of Integers

ℝ - Set of Real Numbers

{ } or ∅ : Empty set (contains no elements)

How to express sets:

**List Notation** {1,2,3,5,14}, {red,green,blue} – separate elements by commas

**Set builder notation**:

{y : y ∈ ℝ and y<=100} “The set of those y such that y is in set ℝ and y<=100”

[Use to easily express large sets]

Can also use variations : { z ∈ ℝ : z<1000},{ 2a+1 : a ∈ ℝ } – [ ‘element of set’ before, ‘such that’ condition on the right]

**Predicate** – declarative statement whose truth values depends on at least one variable

‘x is tall’, ‘y is an athlete’, ‘z<=100’,’w likes t’ [w likes t depends on two variables]

Substitute values for free variables to get a **proposition**

Ie: Q(x) denotes x > 15 [this is the predicate], Q(3)=F, Q(20)=T [those are propositions, q(x) changed for a different value of x]

**Universal Quantification: ∀ means ‘for all’ [**Describes all elements of a set]

Written as: **∀x ∈ *U*, p(x)** – Translates: For all elements, x, in the set *U* , p(x) is true

[ comma separates the ‘for all’ and ‘elements in set’ from the condition that they must satisfy ]

The statements is true **if and only if** for every value *v* in U *,* p(v) is true

[ It is like and’ing the condition applied to all elements of the set ]

If X={1,2,3,4,5} 🡪 ∀x ∈ *S*, C(x) **is true if and only if** C(1) ∧C(2) ∧C(3) ∧C(4)∧C(5) **is true**

**Existential Quantification: ∃ means ‘there exists’**

[ describes something about at least one element in the set ]

Written as: **∃x ∈ *U* such that p(x)** – Translates: There exists an x in the set U such that p(x) is true

True if and only if there is one value *v* in U that satisfies the condition

If X = {1,2,3} 🡪 ∃x ∈ *S*, C(x) **is true if and only if** C(1) ∨ C(2) ∨ C(3) **is true**

[only has to apply to ONE value in the set, *might* apply to more]

**Universals and Existentials are Duals:**

¬**(∀x ∈ *U*, p(x)** **)** **≡ ∃x ∈ *U* such that** ¬ **p(x)**

**∀x ∈ *U*, ¬p(x)** **≡ ¬(∃x ∈ *U* such that p(x))**

**Check slides about common idioms / nested quantifiers**

**Translating with Quantifiers (The ‘take a break from the slides’ parts)**

* **Logic to English**

Let U be the set of all people

E(x) = ‘x is an engineer’

F(x) = ‘x speaks French’

M(x) = ‘x is in the military’

1) Statement: ∀t ∈ *U*, ((E(t) ∧ M(t)) → F(t))

Translation: All engineers in the military speak French

2) S: ∃t ∈ *U* such that (M(t) ∧ ¬F(t))

T: There is at least one person who is in the military and doesn’t speak French.

3) S: ∀t ∈ *U*, (F(t) → E(t))

T: Every person is a French speaking engineer.

4) S: ∃t ∈ *U* such that (E(t) → M(t))

T: There is at least one person who is either a non-engineer or in the military (or both)

* **English to Logic**

Let U be the set of all people

H(w) = ‘w plays hockey’

S(w) = ‘w skates well’

T(w) = ‘w has a full set of teeth’

R(w) = ‘w is a hockey referee’

1) S: Not everyone who plays hockey skates well

T: ¬ (∀z ∈ *U*, (H(z) → S(z)))

2) S: At least one hockey player has a full set of teeth

T: ∃z ∈ *U* such that (H(z) ∧ T(z))

3) S: No hockey referee has a full set of teeth

T: ∀w ∈ *U*, (R(w) → ¬ T(z)))

4) S: Hockey referees play hockey if and only if they skate well

T: ∀y ∈ *U*, (R(y) → (H(y) ↔S(y)))

5) S: Those who play Hockey and those who referee hockey are two distinct groups

T: ∀z ∈ *U*, (H(z) ↔ ¬ R(z))

**Universal Instantiation**

If these premises are true:

∀z ∈ *U*, p(z)

a ∈ *U*

Then this conclusion is true:

p(a)

**Universal Generalization**

Steps:

1. Take an arbitrary element, a ∈ *U*
2. Establish that p(a) holds

Then the conclusion

∀x ∈ *U*, p(x)

Can be obtained.

*An Arbitrary element* means that we need to introduce a name (*a* )as a generic element of U and show that p(a) is true to establish that p(x) is true for all x.

In a proof this step is “Let v ∈ *U* be arbitrary -- Assumption”

*Note: The ‘Bogus’ proofs in the notes highlight what NOT to do*